Chapter 2

Theoretical Basis for Movable Boundary Calculations

2.1 Overview of Approach and Capabilities

This chapter presents the theories and concepts embodied in HEC-6. Information regarding implementation of these theories and concepts in HEC-6 is presented in Chapter 3.

2.1.1 General

HEC-6 processes a discharge hydrograph as a sequence of steady flows of variable durations. Using continuity of sediment, changes are calculated with respect to time and distance along the study reach for the following: total sediment load, volume and gradation of sediment that is scoured or deposited, armoring of the bed surface, and the cross section elevations. In addition, sediment outflow at the downstream end of the study reach is calculated. The location and amount of material to be dredged can be obtained if desired.

2.1.2 Geometry

Geometry of the river system is represented by cross sections which are specified by coordinate points (stations and elevations) and the distances between cross sections. HEC-6 raises or lowers cross section elevations to reflect deposition and scour. The horizontal locations of the channel banks are considered fixed and the floodplains on each side of the channel are considered as having fixed ground locations; however, they will be moved vertically if they are within the movable bed limits specified by the user.

2.1.3 Hydraulics and Hydrology

The water discharge hydrograph is approximated by a sequence of steady flow discharges, each of which continues for a specified period of time. Water surface profiles are calculated for each flow using the standard-step method to solve the energy and continuity equations. Friction loss is calculated by Manning's equation and expansion and contraction losses are calculated if the loss coefficients are specified. Hydraulic roughness is described by Manning's *n* values and can vary from cross section to cross section. At each cross section *n* values may vary vertically or with discharge.

The downstream water surface elevation must be specified for subcritical water surface profile calculations. In the case of a reservoir the operating rule may be utilized, but if open river conditions exist, a stage-discharge rating curve is usually specified as the downstream boundary condition. A boundary condition or operating rule may be used at any location along the main stem or tributaries.

2.1.4 Sediment Transport

Inflowing sediment loads are related to water discharge by sediment-discharge curves for the upstream boundaries of the main stem, tributaries and local inflow points. For realistic computation of stream behavior, particularly scour and stable conditions, the gradation of the material forming the stream bed must be measured. HEC-6 allows a different gradation at each cross section. If only deposition is expected, the gradation of material in the bed is less important.

Sediment gradations are classified by grain size using the American Geophysical Union scale. HEC-6 will compute transport potential for clay (particles less than 0.004 mm diameter), four classes of silt (0.004-0.0625 mm), five classes of sand (from very fine sand, 0.0625 mm, to very coarse sand, 2.0 mm), five classes of gravel (from very fine gravel, 2.0 mm, to very coarse gravel, 64 mm), two class of cobbles (from small, 64mm, to large cobbles, 256mm) and three classes of boulders (from small, 256mm, to large boulders, 2048mm).

Transport potential is calculated at each cross section using hydraulic information from the water surface profile calculation (e.g., width, depth, energy slope, and flow velocity) and the gradation of bed material. Sediment is routed downstream after the backwater computations are made for each successive discharge (time step).

2.2 Theoretical Basis for Hydraulic Calculations

The basis for water surface profile calculations is essentially Method II, which is described in "Backwater Curves in River Channels," EM 1110-2-1409 (USACE 1959). Conveyance is calculated from average areas and average hydraulic radii for adjacent cross sections.

2.2.1 Equations for Water Surface Profile Calculations

The hydraulic parameters needed to calculate sediment transport potential are velocity, depth, width and energy slope - all of which are obtained from water surface profile calculations. The one-dimensional energy equation (Equation 2-1) is solved using the standard step method and the hydraulic parameters are calculated at each cross section for each successive discharge. Figure 2-1 shows a representation of the terms in the energy equation.

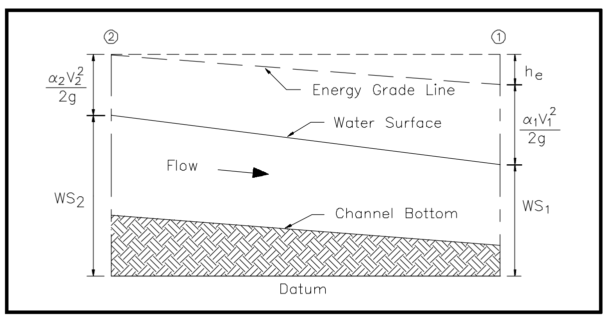


Figure 2-1
Energy Equation Terms

$$WS_2 + \frac{\alpha_2 V_2^2}{2g} = WS_1 + \frac{\alpha_1 V_1^2}{2g} + h \qquad (2\text{-}1)$$

where:

g	=	acceleration of gravity
h_e	=	energy loss
V_1, V_2	=	average velocities (total discharge ÷ total flow area) at ends of reach
WS_1, WS_2	=	water surface elevations at ends of reach
α_1, α_2	=	velocity distribution coefficients for flow at ends of reach.

2.2.2 Hydraulic Losses

2.2.2.1 Friction Losses

River geometry is specified by cross sections and reach lengths; friction losses are calculated by Method II (USACE 1959). The energy loss term, $\mathbf{h_e}$, in Equation 2-1 is composed of friction loss, $\mathbf{h_f}$, and form losses, $\mathbf{h_o}$, as shown in Equation 2-2. Only contraction and expansion losses are considered in the geometric form loss term.

$$h_e = h_f + h_o \qquad (2\text{-}2)$$

To approximate the transverse distribution of flow, the river is divided into strips having similar hydraulic properties in the direction of flow. Each cross section is subdivided into portions that are referred to as subsections. Friction, $\mathbf{h_f}$, loss is calculated as shown below:

$$h_f = \left[\frac{Q}{K_t^{\prime}}\right]^2 \qquad (2\text{-}3)$$

in which:

$$K_t^{\prime} = \sum_{j=1}^{NSS}\left[\frac{1.49}{n_j}\right]\frac{\dfrac{(A_2 + A_1)_j}{2}\left[\dfrac{R_2 + R_1}{2}\right]_j^{2/3}}{L_j^{1/2}} \qquad (2\text{-}4)$$

where:

$\mathbf{A_1, A_2}$	=	downstream and upstream area, respectively, of the flow normal to the cross sections
NSS	=	total number of subsections across each cross section
$\mathbf{K'_t}$	=	length-weighted subsection conveyance
$\mathbf{L_j}$	=	length of the j^{th} strip between subsections
n	=	Manning's roughness coefficient
Q	=	water discharge
$\mathbf{R_1, R_2}$	=	downstream and upstream hydraulic radius, respectively.

2.2.2.2 Other Losses

Energy losses due to contractions and expansions are computed by the following equation:

$$h_o = C_L \left| \frac{\alpha_2 V_2^2}{2g} - \frac{\alpha_1 V_1^2}{2g} \right| \qquad (2\text{-}5)$$

where: $\mathbf{C_L}$ = loss coefficient for expansion or contraction

If the quantity within the absolute value notation is negative, flow is contracting and $\mathbf{C_L}$ is the coefficient of contraction; if it is positive, flow is expanding and $\mathbf{C_L}$ is the coefficient of expansion.

2.2.3 Computation of Hydraulic Elements

Each cross section is defined by coordinates (X,Y) as shown in Figure 2-2. For convenience of assigning *n* values, reach lengths, etc., each cross section is divided into subsections, usually consisting of a main channel, with left and right overbanks.

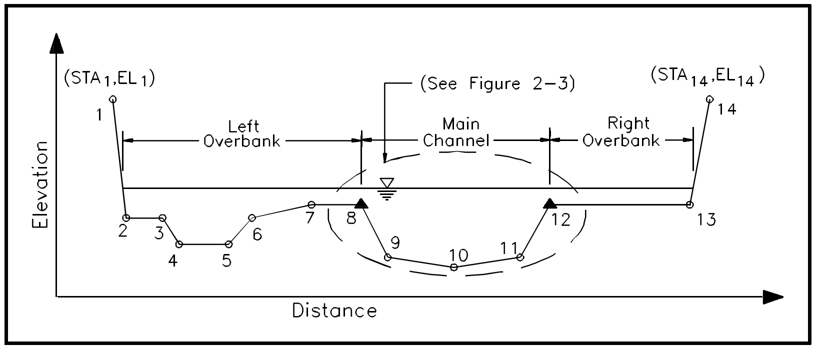


Figure 2-2
Typical Representation of a Cross Section

2.2.3.1 Subsection Area

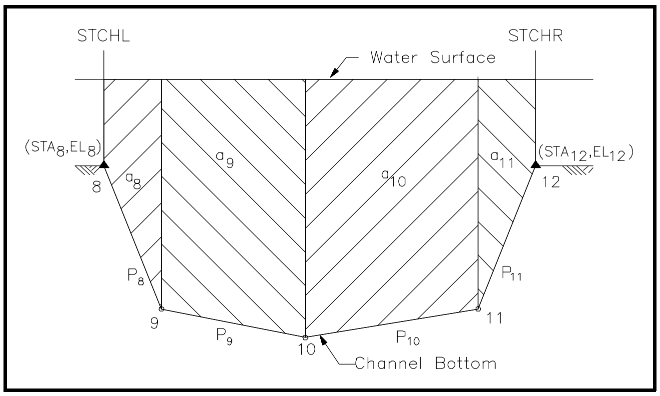


Figure 2-3
Incremental Areas in Channel Subsection

The area of each subsection is computed by summing incremental areas below the water surface between consecutive coordinates of the cross section. Figure 2-3 illustrates the technique with a subsection of Figure 2-2 where STCHL and STCHR are the lateral boundaries of the subsection.

The area of the channel subsection is:

$$A_j = a_8 + a_9 + a_{10} + a_{11} \qquad (2\text{-}6)$$

where: $\mathbf{a_i}$ = incremental area.

The equation for an incremental area, $\mathbf{a_i}$, is:

$$a_i = \frac{(d_i + d_{i+1})\, W}{2} \qquad (2\text{-}7)$$

where: $\mathbf{d_i}$, $\mathbf{d_{i+1}}$ = the left and right depth of each incremental area, respectively (see Figure 2-4)
W = width of an incremental area.

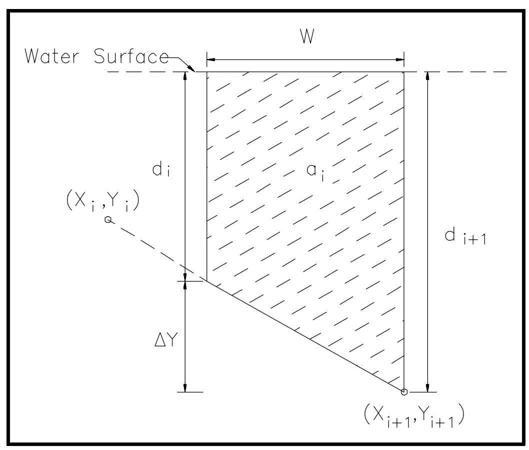


Figure 2-4
Incremental Area

Normally, $\mathbf{d_i}$, $\mathbf{d_{i+1}}$ and **W** are defined by two consecutive cross section coordinate points, as shown in Figure 2-4. However at the first and last increments in each subsection, a subsection station defines one side of the incremental area. If the subsection station does not coincide with an **X** coordinate, straight line interpolation is used to compute the length of either, $\mathbf{d_i}$, $\mathbf{d_{i+1}}$, or both.

2.2.3.2 Wetted Perimeter

The wetted perimeter, **P**, is computed as the length of the cross section below the water surface. In the case of Figure 2-3, this is:

$$P = P_8 + P_9 + P_{10} + P_{11} \tag{2-8}$$

where: **P_i** = incremental wetted perimeter.

The equation for the wetted perimeter of the incremental area in Figure 2-4 is:

$$P_i = (\Delta Y^2 + W^2)^{1/2} \tag{2-9}$$

where: **ΔY** and **W** are as shown in Figure 2-4.

Note that only the distance between coordinate points is considered in p_i, not the depths d_i and d_{i+1}. In other words, friction due to shear forces between subsections is not considered.

2.2.3.3 Hydraulic Radius

The hydraulic radius, **R**, is calculated for each subsection, $_j$, by:

$$R_j = \frac{A_j}{P_j} \tag{2-10}$$

where:
A_j = area of subsection
P_j = wetted perimeter of subsection
R_j = hydraulic radius of subsection.

2.2.3.4 Conveyance

The conveyance, **K_j**, is computed for each subsection, $_j$, by:

$$K_j = \frac{1.49}{n_j} A_j R_j^{2/3} \tag{2-11}$$

The total conveyance, **K_t**, in the cross section is:

$$K_t = \sum_{j=1}^{NSS} K_j \tag{2-12}$$

where: **NSS** = total number of subsections.

2.2.3.5 Velocity Distribution Factor, Alpha

Alpha is an energy correction factor to account for the transverse distribution of velocity across the floodplains and channel. Large values of alpha (>2) will occur if the depth of flow on the overbanks is shallow, the conveyance is small, and the area is large. Alpha is computed as follows:

$$\alpha = \frac{\sum_{j=1}^{NSS} \left[\frac{K_j^3}{A_j^2}\right]}{\left[\frac{K_t^3}{A_t^2}\right]} \tag{2-13}$$

2.2.3.6 Effective Depth and Width

The sediment transport capacity for non-rectangular sections is calculated using a weighted depth, **EFD**, called the effective depth. The corresponding effective width, **EFW**, is calculated from the effective depth to preserve $A(D^{2/3})$ for the cross section.

$$EFD = \frac{\sum_{i=1}^{i_t} D_{avg} \cdot a_i \cdot D_{avg}^{2/3}}{\sum_{i=1}^{i_t} a_i \cdot D_{avg}^{2/3}} \tag{2-14}$$

$$EFW = \frac{\sum_{i=1}^{i_t} a_i \cdot D_{avg}^{2/3}}{EFD^{5/3}} \tag{2-15}$$

where: $\mathbf{a_i}$ = flow area of each trapezoidal element
$\mathbf{D_{avg}}$ = average water depth of each trapezoidal element
$\mathbf{i_t}$ = the total number of trapezoidal elements in a subsection

The sediment transport computation is based upon hydraulics of the main channel only; therefore, the hydraulic elements are from the geometry within the channel limits only.

2.2.3.7 Critical Depth Calculations

To assess if the backwater profiles remain above critical depth, the critical section factor, **CRT**, is computed using Equation 2-16, and compared with the computed section factor at each cross section.

$$CRT = \frac{Q}{\left(\frac{g}{\alpha}\right)^{1/2}} \tag{2-16}$$

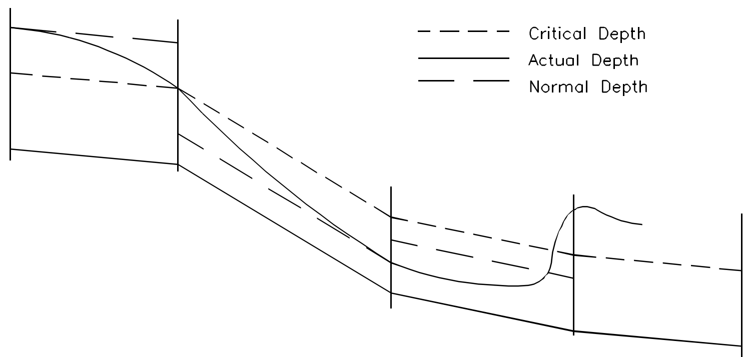
A computed section factor, **ZSQ**, is calculated for comparison to **CRT**.

$$ZSQ = A_t \left(\frac{A_t}{W_t}\right)^{1/2} \tag{2-17}$$

where: $\mathbf{A_t}$ = total area of cross section
$\mathbf{W_t}$ = total water surface width

If **CRT** is less than **ZSQ**, subcritical flow exists and computations continue. Otherwise, critical depth is calculated by tracing the specific energy curve to the elevation of minimum total energy and the resulting water surface elevation is compared with the water surface elevation calculated by Equation 2-1 to decide if flow is supercritical. If supercritical flow is indicated, flow depth is determined as described in the following section.

Hydraulic Properties Used For
Transport Calculation



Trial 4, etc.: This process continues until the assumed and computed values of water surface elevation are within the allowable error tolerance. If they are, the computed water surface elevation becomes the converged solution.

Oscillation between positive and negative "error" is permitted. A note is printed in the event a solution is "forced" (after 20 trials) even though the "error" is greater than the allowable error. In this case, the last computed water surface elevation is used.

2.2.4 Representative Hydraulic Parameters Used in Sediment Calculations

Hydraulic parameters are converted into representative (weighted) values for each reach prior to calculating transport capacity. General equations are shown below. These weighting factors can be modified with input data.

Interior Point (section)

$$VEL = XID \cdot VEL(K-1) + XIN \cdot VEL(K) + XIU \cdot VEL(K+1) \tag{2-18}$$

$$EFD = XID \cdot EFD(K-1) + XIN \cdot EFD(K) + XIU \cdot EFD(K+1) \tag{2-19}$$

$$EFW = XID \cdot EFW(K-1) + XIN \cdot EFW(K) + XIU \cdot EFW(K+1) \tag{2-20}$$

$$SLO = 0.5 \cdot [SLO(K) + SLO(K+1)] \tag{2-21}$$

Upstream Point (section)

$$VEL = UBN \cdot VEL(K) + UBI \cdot VEL(K-1) \tag{2-22}$$

$$EFD = UBN \cdot EFD(K) + UBI \cdot EFD(K-1) \tag{2-23}$$

$$EFW = UBN \cdot EFW(K) + UBI \cdot EFW(K-1) \tag{2-24}$$

$$SLO = SLO(K) \tag{2-25}$$

Downstream Point (section)

$$VEL = DBN \cdot VEL(K) + DBI \cdot VEL(K+1) \tag{2-26}$$

$$EFD = DBN \cdot EFD(K) + DBI \cdot EFD(K+1) \tag{2-27}$$

$$EFW = DBN \cdot EFW(K) + DBI \cdot EFW(K+1) \tag{2-28}$$

$$SLO = SLO(K) \tag{2-29}$$

where:

DBN, DBI = coefficients for downstream reach boundary
K-1, K, K+1 = downstream, midpoint, and upstream locations, respectively, of a reach
SLO = friction slope
UBN, UBI = coefficients for upstream reach boundary
VEL = weighted velocity of the reach
XID, XIN, XIU = downstream, interior, and upstream coefficients, respectively, for interior points.

Several different weighting factors were investigated during the formulation of the computation scheme. Table 2-1 shows the set of factors which appeared to give the most stable calculation and thereby permits the longest time steps (Scheme 1) and the set which is the most sensitive to changes in bed elevation but requires shorter time steps to be stable (Scheme 2). Scheme 1 is often the best choice because the computed energy slope may vary drastically from section-to-section whereas the actual river's behavior may be dependent upon reach properties. HEC-6 defaults to Scheme 2 but this can be changed by entering other values for the weighting factors on the **I5** record.

Table 2-1.
Representative Hydraulic Parameter Weighting Factors

	DBI	DBN	XID	XIN	XIU	UBI	UBN	
Scheme 1	0.5	0.5	0.25	0.5	0.25	0.0	1.0	Most Stable
Scheme 2	0.0	1.0	0.0	1.0	0.0	0.0	1.0	Most Sensitive

2.2.5 Hydraulic Roughness

Boundary roughness of an alluvial stream is closely tied to sediment transport and the movement of bed material. Energy losses for water surface profile calculations must include the effects of all losses: grain roughness of the movable bed, drag losses from bed forms such as ripples and dunes, bank irregularities, vegetation, contraction/expansion losses, bend losses, and junction losses. All these losses except the contraction/expansion losses are embodied in a single roughness parameter, Manning's *n*.

2.3 Theoretical Basis for Sediment Calculations

Sediment transport rates are calculated for each flow in the hydrograph for each grain size. The transport potential is calculated for each grain size class in the bed as though that size comprised 100% of the bed material. Transport potential is then multiplied by the fraction of each size class present in the bed at that time to yield the transport capacity for that size class. These fractions often change significantly during a time step, therefore an iteration technique is used to permit these changes to effect the transport capacity. The basis for adjusting bed elevations for scour or deposition is the Exner equation (see Section 2.3.1.3).

2.3.1 Equation for Continuity of Sediment Material

2.3.1.1 Control Volume

Each cross section represents a control volume. The control volume width is usually equal to the movable bed width and its depth extends from the water surface to the top of bedrock or other geological control beneath the bed surface. In areas where no bedrock exists, an arbitrary limit (called the "model bottom") is assigned (see Figure 2-7).

The control volume for cross section 2 is represented by the heavy dashed lines. The control volumes for cross sections 1 and 3 join that for cross section 2, etc.

The sediment continuity equation is written for this control volume; however, the energy equation is written between cross sections. Because descriptions of both sediment continuity and conservation of energy should enclose the same space; and because the averaging of two cross sections tends to smooth numerical results, the shape of the control volume is conceptually deformed.

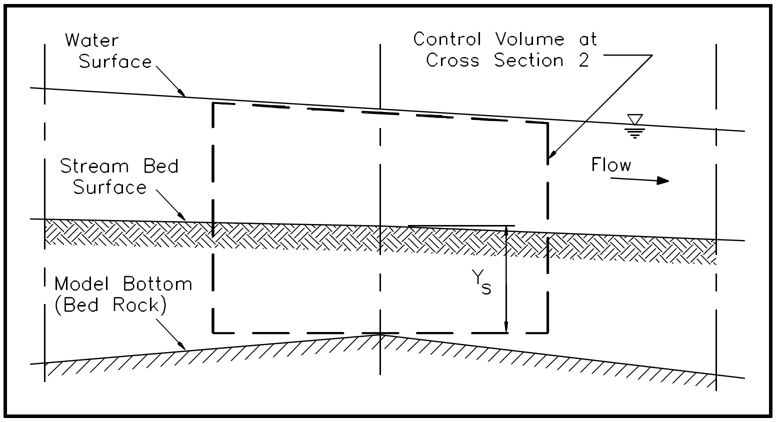


Figure 2-7
Control Volume for Bed Material

The amount of sediment in the stream bed, using an average end area approximation, is:

$$V_{sed} = B_o \cdot Y_s \cdot \frac{L_u + L_d}{2} \qquad (2\text{-}30)$$

where:
- $\mathbf{B_o}$ = width of the movable bed
- $\mathbf{L_u}$, $\mathbf{L_d}$ = length of the upstream and downstream reach, respectively, used in control volume computation
- $\mathbf{V_{sed}}$ = volume of sediment in control volume
- $\mathbf{Y_s}$ = depth of sediment in control volume.

For a water depth, **D**, the volume of fluid in the water column is:

$$V_f = B_o \cdot D \cdot \frac{L_u + L_d}{2} \qquad (2\text{-}31)$$

$\mathbf{B_o}$ and **D** are hydraulic parameters, width and depth, which are calculated by averaging over the same space used in solving the energy equation as described in Sections 2.2.1 and 2.2.4.

The solution of the continuity of sediment equation assumes that the initial concentration of suspended bed material is negligible. That is, all bed material is contained in the sediment reservoir at the start of the computation interval and is returned to the sediment reservoir at the end of the computation interval. Therefore, no initial concentration of bed material load need be specified in the control volume.

The hydraulic parameters, bed material gradation and calculated transport capacity are assumed to be uniform throughout the control volume. The inflowing sediment load is assumed to be mixed uniformly with sediment existing in the control volume. HEC-6 assumes instantaneous diffusion of all grain size classes on a control volume basis.

2.3.1.2 Concepts of the Control Volume

The control volume concept employed in HEC-6 represents the alluvium of a natural river. Over time, the river will exchange sediment with its boundaries both vertically and laterally, changing its shape by forming channels, natural levees, meanders, islands, and other plan forms. HEC-6, however, only models vertical sediment exchange with the bed; the width and depth of which are user defined. Correct reproduction of the natural river system depends on modeling the proper exchange of sediment between the flow field and the bed sediment. The physics of that exchange process are not well understood.

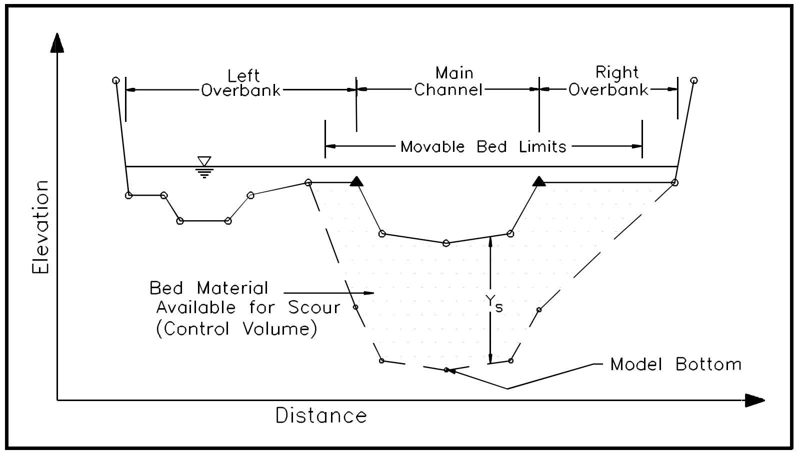
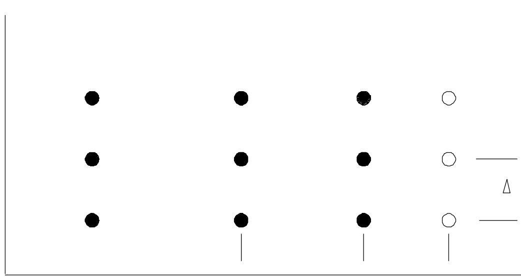


Figure 2-8
Sediment Material in the Streambed



Equations 2-33 and 2-34 represents the Exner Equation expressed in finite difference form for point P using the terms shown in Figure 2-9.

$$\frac{G_d - G_u}{0.5(L_d + L_u)} + \frac{B_{sp}(Y_{sp}^{\prime} - Y_{sp})}{\Delta t} = 0 \qquad (2\text{-}33)$$

$$Y_{sp}^{\prime} = Y_{sp} - \frac{\Delta t}{(0.5)B_{SP}} \cdot \frac{G_d - G_u}{L_d + L_u} \qquad (2\text{-}34)$$

where:

$\mathbf{B_{sp}}$	=	width of movable bed at point P
$\mathbf{G_u}$, $\mathbf{G_d}$	=	sediment loads at the upstream and downstream cross sections, respectively
$\mathbf{L_u}$, $\mathbf{L_d}$	=	upstream and downstream reach lengths, respectively, between cross sections
$\mathbf{Y_{sp}}$, $\mathbf{Y'_{sp}}$	=	depth of sediment before and after time step, respectively, at point P
0.5	=	the "volume shape factor" which weights the upstream and downstream reach lengths
Δt	=	computational time step

The initial depth of bed material at point P defines the initial value of $\mathbf{Y_{sp}}$. The sediment load, $\mathbf{G_u}$, is the amount of sediment, by grain size, entering the control volume from the upstream control volume. For the upstream-most reach, this is the inflowing load boundary condition provided by the user. The sediment leaving the control volume, $\mathbf{G_d}$, becomes the $\mathbf{G_u}$ for the next downstream control volume.

The sediment load, $\mathbf{G_d}$, is calculated by considering the transport capacity at point P, the sediment inflow, availability of material in the bed, and armoring. The difference between $\mathbf{G_d}$ and $\mathbf{G_u}$ is the amount of material deposited or scoured in the reach labelled as "computational region" on Figure 2-9, and is converted to a change in bed elevation using Equation 2-34.

The transport potential of each grain size is calculated for the hydraulic conditions at the beginning of the time interval and is not recalculated during that interval. Therefore, it is important that each time interval be short enough so that changes in bed elevation due to scour or deposition during that time interval do not significantly influence the transport potential by the end of the time interval. Fractions of a day are typical time steps for large water discharges and several days or even months may be satisfactory for low flows. The amount of change in bed elevation that is acceptable in one time step is a matter of judgment. Good results have been achieved by using either 1 ft or 10% of the water depth, whichever is less, as the allowable bed change in a computational time interval. The gradation of the bed material, however, **is** recalculated during the time interval because the amount of material transported is very sensitive to the gradation of bed material.

2.3.1.4 Bed Gradation Recomputations

HEC-6 solves the Exner equation for continuity of sediment. If transport capacity is greater than the load entering the control volume, available sediment is removed from the bed to satisfy continuity. Since transport capacity for a given size depends upon the fraction of that size on the bed, it is necessary to frequently recalculate fractions present as sediment is exchanged with the bed. The number of exchange increments, **SPI,** during a time step is theoretically related to the time step length, **Δt**, velocity, and reach length in each reach by:

$$NO.\ OF\ EXCHANGE\ INCREMENTS = \frac{\Delta t \cdot VELOCITY}{REACH\ LENGTH} \tag{2-35}$$

Usually the number of exchange increments can be less than this without generating significant numerical problems. Specify SPI in field 2 of the **I1** record. Initially, SPI should be set to zero (which invokes Equation 2-35) and an extreme hydrologic event simulated. This should be the most stable (and computationally intensive) case. Then, starting from SPI=50 or more, one should decrease it in increments of 10 until the results become significantly different from the results with SPI=0. Use the smallest SPI that gives a solution close to that obtained with SPI=0.

2.3.2 Determination of the Active and Inactive Layers

HEC-6 implements the concept of an active and an inactive bed layer. The active layer is assumed to be continually mixed by the flow, but it can have a surface of slow moving particles that shield the finer particles from being entrained in the flow. Two different processes are simulated: (1) Mixing that occurs between the bed sediment particles and the fluid-sediment mixture due to the energy in the moving fluid and, (2) Mixing that occurs between the active layer and the inactive layer due to the movement of the bed surface. The mixing mechanisms are attributed to large scale turbulence and bed shear stress from the moving water. The mixing depth (termed "equilibrium depth") is expressed as a function of flow intensity (unit discharge), energy slope, and particle size.

2.3.2.1 Equilibrium Depth

The minimum energy hydraulic condition at which a particular grain size will just be stationary on the bed surface can be calculated by combining Manning's, Strickler's, and Einstein's equations, respectively:

$$V = \frac{1.49}{n} R^{2/3} S_f^{1/2} \tag{2-36}$$

$$n = \frac{d^{1/6}}{29.3} \tag{2-37}$$

$$\psi = \frac{\rho_s - \rho_f}{\rho_f} \cdot \frac{d}{DS_f} \tag{2-38}$$

where:
- **d** = grain diameter
- **D** = water depth
- **V** = water velocity
- ρ_s = density of sand grains
- ρ_f = density of water
- ψ = transport intensity from Einstein's bed load function, related to the inverse of Shield's parameter
- S_f = friction slope

For negligible transport, ψ equals 30 or greater. Solving Equation 2-38 in terms of S_f for a specific gravity of sand of 2.65 and with ψ set at 30 yields:

$$S_f = \frac{d}{18.18D} \tag{2-39}$$

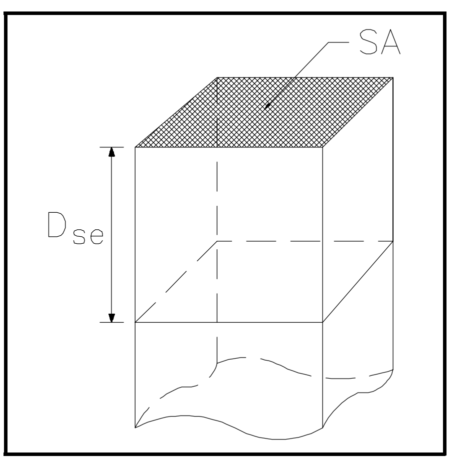


Figure 2-10
A Column of Bed Material Having Surface Area (SA)

The surface area of the column may be partially shielded by a rock outcrop or an armor layer such that the potential scour area is less than the total surface area of the column. This reduces the number of grains, **N**, exposed to scour as follows:

$$N = \frac{A \cdot SAE}{\left[\frac{\pi d^2}{4}\right]} \tag{2-44}$$

where: **SAE** = ratio of surface area of potential scour to total surface area

Assuming a mixture of grain sizes, the depth of scour required to produce the volume of a particular grain size that is sufficient to completely cover the bed to a thickness of one grain diameter is:

$$V_{se} = PC \cdot SA \cdot D_{se} = N \frac{\pi d_a^3}{6} \tag{2-45}$$

where:
- $\mathbf{d_a}$ = smallest stable grain size in armor layer
- $\mathbf{D_{se}}$ = depth of bed material which must be removed to reach equilibrium in a time step
- **PC** = fraction of bed material coarser than size d_a
- $\mathbf{V_{se}}$ = volume of bed material which must be removed to reach equilibrium in a time step

Combining the surface area and volume equations and solving for the required depth of scour to fully develop the armor layer gives:

$$D_{se} = \left[\frac{SA \cdot SAE}{(\pi d^2/4)}\right] \cdot \left[\frac{(\pi d^3/6)}{PC \cdot SA}\right] = \left(\frac{2}{3}\right)\left[\frac{SAE \cdot d}{PC}\right] \tag{2-46}$$

This equation is used with Equation 2-41 to calculate an equilibrium depth for a mixture of grain sizes. In order to determine the **PC** to use in Equation 2-46, the proper segment on the bed gradation curve is found by approximating the functional relationship between **d** and **PC** with a sequence of straight line segments as shown in Figure 2-11. The first step in locating the proper segment on the gradation curve is to calculate the equilibrium depths, $\mathbf{D1_{eq}}$ and $D2_{eq}$ for the grain sizes at points 1 and 2 (Figure 2-12) using Equation 2-41. If the actual water depth, D_W, is less than $D2_{eq}$, the straight line segment from 1 to 2 in Figure 2-11 defines the required functional relationship and the final equilibrium depth is calculated. If D_W is greater than the equilibrium depth for grain size at point 2, computations move down the gradation curve to points 2 to 3, 3 to 4, etc., until either the proper segment is located or the smallest grain size is sufficient to armor the bed in which case scour will not occur.

HEC-6 designates the zone of material between the bed surface and equilibrium depth as the active layer and the zone from equilibrium depth to the model bottom as the inactive layer. The active layer provides the source of material forming the bed surface. The inactive layer initially has the same gradation as the parent bed. That gradation changes as material is deposited on the active layer and is exchanged with the inactive layer. Material is moved from one layer to the other layer as the active layer thickness changes with water depth, velocity and slope. Only the material in the active layer is subject to scour. HEC-6 allows sorting by grain size during the solution of the Exner equation which requires continuous accounting of the percent of sediment in each size class within each time step. When all material is removed from the active layer, the bed is completely armored for that hydraulic condition.

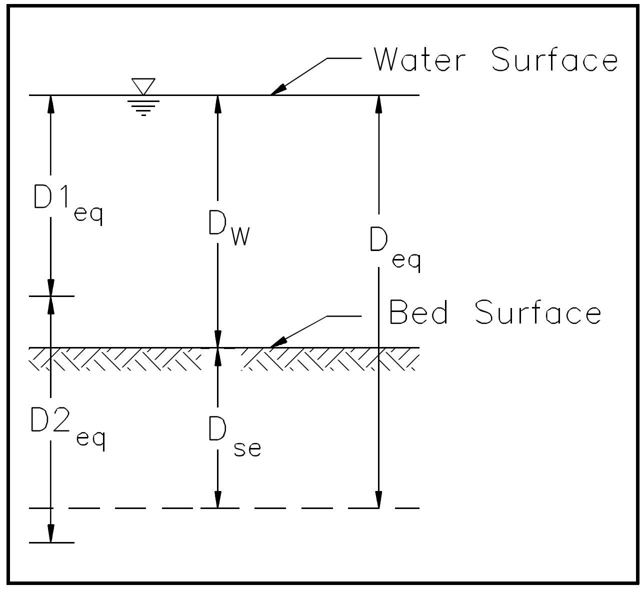


Figure 2-12
Equilibrium Depth Conditions

Assuming that the bed material is well mixed the rate of armoring is proportional to the volume of material removed, and the surface area exposed, **SAE**, for scour is:

$$SAE = \frac{VOL_A}{VOL_{SE}} \tag{2-47}$$

where: **VOL_A** = volume remaining in active layer
VOL_{SE} = total volume in active layer

Leaching of the smaller particles from beneath the bed surface is prevented by adjusting the **SAE**. If a grain of bed sediment is smaller than the armor size, transport capacity is linearly decreased to zero as **SAE** decreases to 40% of the total bed surface (Harrison 1950). Thereafter, only the inflowing load of that grain size and smaller is transported through the reach. Particle sizes equal to and larger than the armor size are not constrained by this procedure.

2.3.3.1 Impact of the Active Layer on Depth of Erosion

After the depth of the active layer has been calculated, Method 1 completes the bed change calculation for that cross section. At each exchange increment (SPI), Method 1 checks the volume of sediment in the active layer. However, if all material has been removed before the last exchange increment of the time step, HEC-6 does not give a warning message. When this happens, the calculated erosion rates and depths will be too small.

To avoid such a condition, the duration of each computation time step must be tested and reduced until further reductions do not change the results. This procedure is similar to the calibration method described in HEC (1992).

2.3.3.2 Composition of the Active Layer

When computations begin, the gradation of the active layer defaults to the inactive layer gradation.

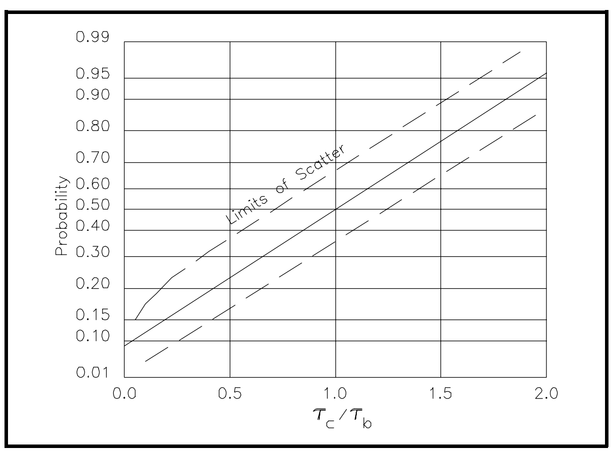


Figure 2-13
Probability of Grain Stability

will remain stationary in the bed. This relationship is used to calculate a bed stability coefficient, **BSF**, which includes the particle size distribution of the active layer as follows:

$$BSF = \frac{\sum_{i=1}^{NGS} PROB \cdot PROB \cdot PI_i \cdot d_{mi}}{\sum_{i=1}^{NGS} PROB \cdot PI_i \cdot d_{mi}} \tag{2-50}$$

where: **d_{mi}** = median grain diameter for grain size class i
i = grain size class analyzed
NGS = number of grain sizes present
PI = fraction of bed composed of a grain size class
PROB = probability that grains will stay in the bed

Gessler (1970) proposed that a stability factor equal to or greater than 0.65 indicates a stable armor layer. If a partially armored bed is stable for a given hydraulic condition, material is taken from the active layer until enough stable grains are left to cover the bed to the depth of one stable grain size. If the armored bed is not stable, the layer is destroyed and a completely new active bed is calculated.

The probability function could be used to determine the am

2.3.3.5 Some Limitations of Method 1

This method for computing hydraulic sorting and armoring has exhibited the following shortcomings:

(1) In rivers with large gradation coefficients it appeared that there was too much leaching of sands; i.e., insufficient "armoring".

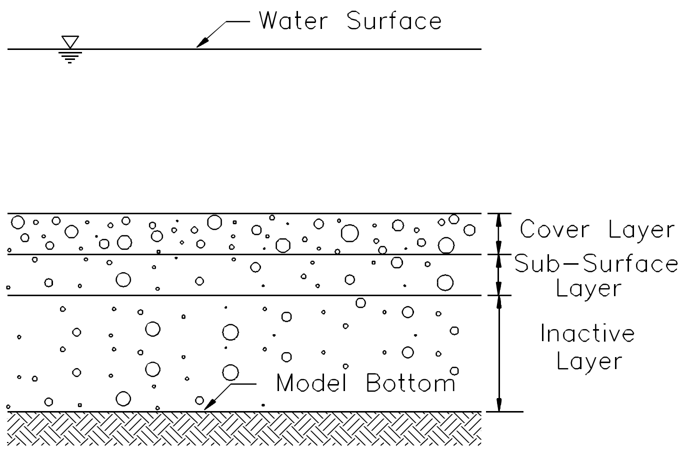
(2) The active layer was too thick in many large sand bed rivers which dampened hydraulic sorting.

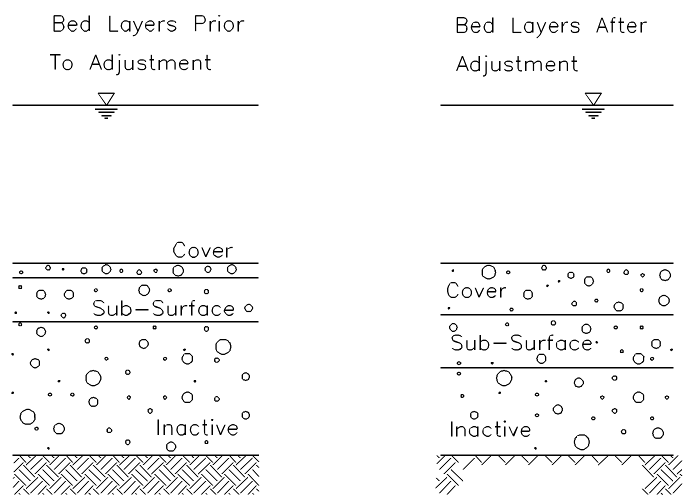
(3) A sediment continuity problem was observed when consolidated silts and clays were exchanged between the active and inactive layers.

2.3.4 Hydraulic Sorting of the Bed Material - Method 2

A second method of computing hydraulic sorting was developed to alleviate some of the limitations of Method 1. This algorithm is based on the concept that exchange of sediment particles occurs within a thin "cover layer" of bed material at the bed surface which is continually mixed by the flow. It is presumed that, as the bed progresses toward an equilibrium condition in which deposition and resuspension of each size class is balanced, the slow moving thin cover layer becomes coarser and serves as a shield, regulating the entrainment of finer particles below. If the cover layer is replenished by deposition from the water column, it will remain as a shield constraining the entrainment of finer material from below. Harrison (1950) observed that this shielding began to occur when as little as 40% of the bed surface was covered. If conditions change such that more material is scoured from, than deposited on, the cover layer; then the cover layer begins to disintegrate and more fine material can be removed from below. Eventually, the cover layer may be completely removed and the bed surface takes on the composition of the material below. This conceptual process replaces the concepts of "surface-area exposed," **SAE**, and "bed-stability factor," **BSF**, used in Method 1.

In Method 2 there are two components of the active layer; a cover layer that is retained from the previous time step and a sub-surface layer that is created at the beginning of the time step from the inactive layer. The sub-surface layer material is returned to the inactive layer at the end of the time step. The cover layer from the previous time step is limited to an arbitrary maximum thickness 2 ft. If the previous cover layer thickness is 2 ft or greater, the new cover layer is assigned a thickness of 0.2 ft (This is approximately equal to the sampling depth of a standard US BM-54 Bed Material Sampler). The residual material is mixed with the inactive layer. The initial thickness of the sub-surface layer is calculated using the equilibrium depth concept presented in Section 2.3.2.1. The maximum thickness, however, is constrained by an estimated maximum scour that could occur during the exchange increment. The estimated maximum scour is calculated from the hydraulics, inactive bed gradation, and selected transport function. This constraint will almost always override the thickness calculated using equilibrium depth. A minimum thickness of two times the largest grain size in transport is also imposed. The computation of bed layer adjustments during a time step using Method 2 is depicted on Figures 2-14 through 2-16.


Water Surface
Cover Layer
Sub-Surface
Layer
Inactive
Layer
Model Bottom


Bed Layers Prior
To Adjustment
Cover
Sub–Surface
Inactive
Bed Layers After
Adjustment
Cover
Sub–Surface
Inactive

2.3.4.2 Characteristic Rate of Entrainment

The characteristic rate of entrainment is associated with flow turbulence. Turbulence simulation, however, is beyond the scope of HEC-6. Since sediment entrainment is not instantaneous, a characteristic "flow-distance" was created to approximate a finite rate of entrainment. Using the distance one would need to sample equilibrium concentrations in a flume as a guide, the characteristic distance for entrainment was set at 30 times the flow depth. The entrainment ratio, **ENTRLR**, associated with the rate at which a flow approaches its equilibrium load, is calculated by dividing the reach length by the characteristic distance for entrainment as follows:

$$ENTRLR = \frac{REACH\ LENGTH}{30 \cdot DEPTH} \tag{2-53}$$

The entrainment coefficient, **ETCON**, is then defined by:

$$ETCON = 1.368 - e^{-ENTRLR} \tag{2-54}$$

ETCON is used to determine what percentage of the equilibrium concentration (for each grain size) is achieved in the reach, and has a maximum of 1.0. Research is needed to substantiate this entrainment hypothesis as well as the appropriate equation and coefficients.

2.3.4.3 Characteristic Rate for Deposition

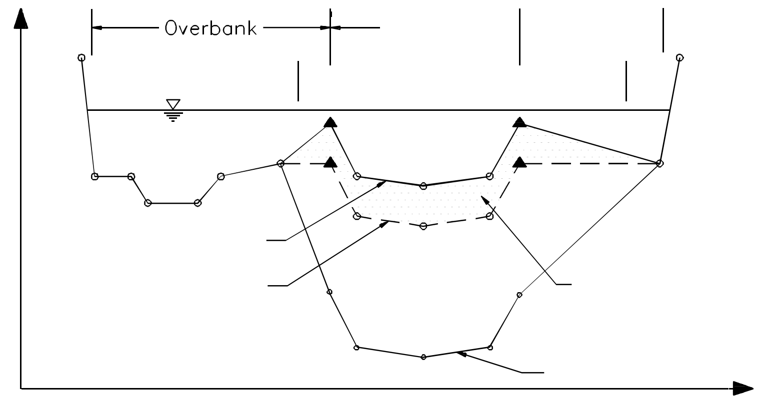
Deposition occurs when the inflowing sediment discharge is greater than the transport capacity. Not all size classes in a mixture will deposit; therefore, this process is calculated by size class. The rate at which sediment deposits from the flow field is controlled by particle settling velocity as follows:

$$DECAY(i) = \frac{V_s(i) \cdot \Delta t}{D_s(i)} \tag{2-55}$$

where: **D_s(i)** = effective depth occupied by sediment size i
Δt = duration of time step
V_s(i) = settling velocity for particle size i

2.3.4.4 Some Limitations of Method 2

In low flow deposition zones, the cover layer becomes the depository for fine materials. In a natural river it is not mixed with sub-surface material; therefore, it retains its fine composition and can be easily removed at high flows. In HEC-6, however, transport capacity is calculated based on the composition of the entire active layer. This probably results in under-prediction of transport capacities for the finest size classes. This may depress the transport of fines, resulting in increased deposition and/or decreased scour. Modifications to the technique of computing **PI_i** for Method 2 may be considered in the future if this becomes a problem. The arbitrary maximum cover layer thickness of 2 ft may hinder deposition during low energy conditions. Mixing of fine material will probably result in underestimation of scour during high flows. Erosion of fine material may be too severely constrained by the Harrison (1950) observation (see Section 2.3.3) which also limits withdrawal from the sub-surface layer.


Overbank

2.3.6 Unit Weight of Deposits

2.3.6.1 Initial Unit Weight

Unit weight is the weight per unit volume of a deposit expressed as dry weight.

$$\gamma_s = (1 - P_d) \cdot SG \cdot \gamma \tag{2-56}$$

where:
- P_d = porosity of deposits
- **SG** = specific gravity of sediment particles
- γ = unit weight of water
- γ_s = unit weight of sediment

Standard field tests are recommended when major decisions depend on the unit weight. Otherwise, use tables on pages 39-41 of "Sedimentation Engineering" (Vanoni 1975) when field data is lacking at your project site.

2.3.6.2 Composite Unit Weight

When dealing with mixtures of particle sizes, the composite unit weight, γ_{SC}, of the mixture is computed using Colby's equation (Vanoni 1975):

$$\gamma_{SC} = \frac{1}{\left[\frac{F_{SA}}{\gamma_{SA}} + \frac{F_{SL}}{\gamma_{SL}} + \frac{F_{CL}}{\gamma_{CL}}\right]} \tag{2-57}$$

where:
- γ_{SA}, γ_{SL}, γ_{CL} = unit weight of sand, silt, and clay, respectively
- F_{SA}, F_{SL}, F_{CL} = fraction of sand, silt, and clay, respectively, in the deposit

2.3.6.3 Consolidated Unit Weight

Compaction of deposited sediments is caused by the grains reorienting and squeezing out the water trapped in the pores. The equation for consolidation (Vanoni 1975) is:

$$\gamma = \gamma_1 + B \cdot \log_{10} T \tag{2-58}$$

where:
- **B** = coefficient of consolidation for silts or clay
- **T** = accumulated time in years
- γ_1 = initial unit weight of the sediment deposit, usually after one year of consolidation

Suggested values of γ_1 and **B** are given on page 43 of Vanoni (1975).

The average consolidated unit weight over a time period **T** requires integration over time. This is computed using the following relationship developed by Miller (1953).

$$\gamma_{ave} = \gamma_1 + B \cdot \left[\frac{T}{T-1}\right] \cdot \log_{10} T - 0.434\ B \tag{2-59}$$

These unit weights are used to convert sediment weight to volume for computation of the bed elevation change.

2.3.7 Sediment Particle Properties

Four basic sediment properties are important in sediment transport prediction: size, shape factor, specific gravity, and fall velocity. Grain size classes are fixed in HEC-6 and described in Section 3.3. The particle shape factor, **SF**, is defined by:

$$SF = \frac{c}{(a \cdot b)^{1/2}} \tag{2-60}$$

where: **a, b, c** = the lengths of the longest, intermediate, and shortest, respectively, mutually perpendicular axes of a sediment particle

The particle shape factor is 1.0 for a perfect sphere and can be as low as 0.1 for very irregularly shaped particles. HEC-6 uses a shape factor default of 0.667 but it can be user specified. If a "sedimentation diameter" is used, which is determined by the particle's fall velocity characteristics, the particle shape factor of 1.0 should be used. If the actual sieve diameter is used, the actual shape factor should be used.

Specific gravity of a particle is governed by its mineral makeup. In natural river systems the bed material is dominated by quartz which has a specific gravity of 2.65. HEC-6 uses 2.65 as a default; however, values of specific gravities for sand, silt, and clay may be input.

Two techniques for calculating particle fall velocity are available in HEC-6. The first is based upon the fall velocities determined by Toffaleti (1966) and is similar to Rubey's method (Vanoni 1975). This method assumes 0.9 as the shape factor. The second, which takes into consideration the particle shape factor, utilizes the procedure described in ICWR (1957), and is described in detail by Williams (1980). The second method is the default.

2.3.8 Silt and Clay Transport

2.3.8.1 Cohesive Sediment Deposition

The equation for silt and clay deposition (Krone 1962) in a recirculating flume at slow aggregation rates and suspended sediment load concentrations less than 300 mg/ℓ is:

$$\ln \frac{C}{C_o} = -k^{\prime} t \tag{2-61}$$

or

$$\frac{C}{C_o} = e^{(-k^{\prime} t)} \tag{2-62}$$

where:
- **C** = concentration at end of time period
- $\mathbf{C_o}$ = concentration at beginning of time period
- **D** = water depth
- **k′** = $\frac{V_s P_r}{2.3D}$
- $\mathbf{P_r}$ = probability that a floc will stick to bed $(1 - \tau_b/\tau_d)$
- **t** = time = reach length/flow velocity
- $\mathbf{V_s}$ = settling velocity of sediment particles
- $\boldsymbol{\tau_b}$ = bed shear stress
- $\boldsymbol{\tau_d}$ = critical bed shear stress for deposition.

This ratio is multiplied by the inflowing clay or silt concentration to obtain the transport potential. The concentration is converted to volume and deposited on the bed.

2.3.8.2 Cohesive Sediment Scour

Erosion is based upon work by Parthenaides (1965) and adapted by Ariathurai and Krone (1976). Particle erosion is determined by:

$$C = \frac{M_1 \cdot S_a}{Q \cdot \gamma} \cdot \left[\frac{\tau_b}{\tau_s} - 1 \right] + C_o \qquad (2\text{-}63)$$

where:
- $\mathbf{C}$ = concentration at end of time period
- $\mathbf{C_o}$ = concentration at beginning of time period
- $\mathbf{M_1}$ = erosion rate for particle scour
- $\mathbf{Q}$ = water discharge
- $\mathbf{S_a}$ = surface area exposed to scour
- τ_b = bed shear stress
- τ_s = critical bed shear for particle scour
- γ = unit weight of water

As the bed shear stress increases, particle erosion gives way to mass erosion and the erosion rate increases. Because the mass erosion rate can theoretically be infinite, Ariathurai and Krone (1976) recommended that a "characteristic time", $\mathbf{T_e}$, be used. With a computation interval of $\mathbf{\Delta t}$, the mass erosion equation becomes:

$$C = \frac{M_2 \cdot S_a}{Q \cdot \gamma} \cdot \frac{T_e}{\Delta t} + C_o \qquad (2\text{-}64)$$

where:
- $\mathbf{\Delta t}$ = duration of time step
- $\mathbf{M_2}$ = erosion rate for mass erosion
- $\mathbf{T_e}$ = characteristic time of erosion

Ariathurai and Krone (1976) give guidance on how to obtain or estimate $\mathbf{T_e}$, $\mathbf{M_1}$, and $\mathbf{M_2}$. Because erosion thresholds and rates for cohesive sediments are dependent on specific sediment particle and ambient water conditions such as mineralogy, sodium adsorption ratio, cation exchange capacity, pH, salinity, and depositional history, in situ and/or laboratory testing are the recommended methods to determine the erosion characteristics of cohesive sediments. A good discussion of cohesive material transport is found in USACE (1991).

2.3.8.3 Influence of Clay on the Active Layer

The presence of clay in the streambed can cause the bed's strength to be greater than the shear stress required to move individual particles. This results in limiting the entrainment rate under erosion conditions. HEC-6 attempts to emulate this process by first checking the percentage of clay in the bed. If more than 10% of the bed is composed of clay, the entrainment rate of silts, sands and gravels is limited to the entrainment rate of the clay. This also prevents the erosion of silts, sands and gravels before the erosion of clay even if the bed shear is sufficient to erode those particles but not enough to erode the cohesive clay.

2.3.8.4 Mudflow Constraint on Transport Potential

Because Einstein's concept of the "equilibrium concentration" is utilized for the