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Subject: Calculation of Size Fractions from PF-Record

Introduction

In non-cohesive sediment transport, there is a linkage between the particle sizes in the mixture being transported and the particle sizes in the bed mixture . Einstein demonstrated the two gradations are entirely different. His transport function supplied the relationship to link them together for the equilibrium condition. He defined the equilibrium condition as that which exists when the same number particles are deposited on the bed as are eroded from it in each size class.

This section describes how the Einstein concept is implemented in HEC-6T. First the section discusses how HEC-6T calculates bed fractions from the input PF-Record. Next, it shows how the Einstein concept, which is for the equilibrium condition, is extended for HEC-6T which is a non-equilibrium sediment transport model.

To purpose is to illustrate how to code input data for HEC-6T. The description uses the example of input and output shown below.

Input Data

```
T4C Carmel River Sediment Inflow and Bed Gradation Data
T5
T6
T7
T8
I1          100       0              1       0       0       0
I4 MPM-T     12       1      14       0       0       0       0       0
I5  STAB    0.5     0.5    0.25     0.5    0.25     0.0     1.0
LQ   cfs     1.     50.    200.    500.   1000.   2000.   4000.   6000.   8500.
LT   tpd  .0001   1017.   7057.  20055.  23276.  27324.  33116.  36987.  43142.
LF   vfs   0.05    0.05    0.05    0.05    0.05    0.05    0.05    0.05    0.05
LF    fs   0.13    0.13    0.13    0.13    0.13    0.13    0.13    0.13    0.13
LF    ms   0.28    0.28    0.28    0.28    0.28    0.28    0.28    0.28    0.28
```

LF cs	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
LF vcs	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
LF vfg	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
LF fg	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
LF mg	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
LF cg	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
LF vcg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LF sc	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LF lc	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LF sb	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LF mb	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PF SAND	0.5	1.0	25.0	12.5	98.9	6.3	94.3	1.97	84.4
PFC 1.37	77.0	0.968	67.6	0.679	56.8	0.479	44.2	0.335	25.4
PFC0.237	16.6	0.168	9.27	0.119	3.70	0.084	0.919	0.062	0.00

Printout Tables from HEC-6T

Intermediate Calculations. A C-Level trace printout was selected for reading the Sedimentary Data Set, see the T4-Record above. The result is a table of the PF-Records as they are read, SED-5.

TABLE SED-5. GRADATION OF BED SEDIMENT RESERVOIR FROM PF-RECORDS.

PF SAND	0.5	1.0	25.0	12.5	98.9	6.3	94.3	1.97	84.4
PFC 1.37	77.0	0.968	67.6	0.679	56.8	0.479	44.2	0.335	25.4
PFC0.237	16.6	0.168	9.27	0.119	3.70	0.084	0.919	0.062	0.001

After the SED-5 table, several tables are printed during the calculation of fractions from the PF-Records. These are described here. The first two rows show GSIZMM which are the class intervals, in mm, selected in fields 3 and 4 of the I4-Record.

GSIZMM =	0.063	0.125	0.250	0.500	1.000	2.000	4.000	8.000	16.000	32.000
GSIZMM =	64.000	128.000	256.000	512.000	1024.000					

Notice that the largest particle in the GSIZEMM table is 1024 mm. This is size class number 14 on the I4-4 Record which translates to Medium Boulders in the grain size table for the I4-Record.

The next four lines of printout show that the values from the PF-Record have been reversed. These are displayed as increasing size of particles rather than decreasing. Particle sizes are on the top two rows, labeled DAXIS, and percent finer values are on the bottom two, labeled PFAXIS.

DAXIS =	25.000	12.500	6.300	1.970	1.370	0.968	0.679	0.479	0.335	0.237
DAXIS =	0.168	0.119	0.084	0.062	0.001					
PFAXIS =	100.000	98.900	94.300	84.400	77.000	67.600	56.800	44.200	25.400	16.600
PFAXIS =	9.270	3.700	0.919	0.000	0.000					

Next, percent finer values are calculated for each class interval, GSIZMM. The rows are labeled PFINER values. There are 15 values providing 14 intervals. These values correlate with the GSIZMM values shown above: 0.0625, 0.125, 0.250 ... etc up to 1024 mm.

```
PFINER =    0.024   4.495  17.958  45.749  68.480  84.529  90.432  95.904  99.292 100.000
          100.000 100.000 100.000 100.000 100.000
```

Results The fraction of sediment in a class is calculated by subtracting the percentages at the boundaries of that class interval and converting the difference to a decimal. Results are labeled PI(_) and are printed from fine to coarse in the following table. For example. The very fine sand class is shown as 0.063 to 0.125 in the GSIZEMM table above. The PFINER values corresponding to that range are 0.024 percent and 4.495 percent. The difference, 4.471 percent, is the amount of very fine sand in the bed mixture. Expressing that value as a fraction, it is 0.04471 which rounds to 0.0447 in the printout table below.

```
ASN      SAE     DMAX     DXPI      XPI    PI( )    PI( )  PI( )    PI( )    PI( )
                                            PI( )    PI( )  PI( )    PI( )    PI( )
    .5  1.0000  0.0820  0.0820  0.0000  0.0447  0.1346  0.2779  0.2273  0.1605
                                        0.0590  0.0547  0.0339  0.0071  0.0000
                                        0.0000  0.0000  0.0000  0.0000

     1  1.0000  0.0820  0.0820  0.0000  0.0447  0.1346  0.2779  0.2273  0.1605
                                        0.0590  0.0547  0.0339  0.0071  0.0000
                                        0.0000  0.0000  0.0000  0.0000
```

There are fourteen size fractions for each cross section. In the PI() columns above, the first three rows are for cross section .5. They begin with Very Fine Sand and continue sequentially to Medium Boulders. Values on the first row are sand: Very Fine Sand, Fine Sand, Medium Sand, Coarse Sand and Very Coarse Sand. These are followed on the second row by the fractions of gravel: PI(Very Fine Gravel) = 0.0590 ... up to PI(Very Coarse Gravel) = 0.0000.

How did the fraction of Very Coarse Gravel in the PI() table get to be zero? It resulted from the PF-Record. The largest grain size on the PF-Record, i.e. the Dmax, is 25 mm. In the grain size table for the I4-Record, twenty-five millimeters falls between 16 mm and 25 mm in the Grain Size Column which is Classified as Coarse Gravel, ID Number 9. Therefore, all particle sizes larger than Coarse Gravel plot on the 100 Percent Finer line on the gradation curve. The percent of sediment particles in these classes is still calculated by subtracting the percents at the class intervals. Consequently, the difference is zero.

Solution of the Sediment Continuity Equation

The size fractions shown in the PI() table above are used in the initial solution of the sediment continuity equation. Following the Einstein concept of sediment transportation, the equation for transport capacity is

$$q_{si} = \sum_{N=1}^{LTI} \sum_{L=1}^{NGS} P_{i,l}\, q_{pl}$$

where

P_i	=	The fraction of each size class, L, on the bed.
q	=	The unit transport capacity for each size class, L, in tons/day/foot of width
LTI	=	The iterations specified for integration of the Exner Equation. This is the SPI value on the I1- Record, Field 2
NGS	=	Total number of Sand and Larger grain sizes in the model, I4-Record

The transport equation is combined with the sediment continuity equation, called the Exner equation. It is

$$\frac{\partial G}{\partial x} + B_o \frac{\partial y_s}{\partial t} = 0$$

where:

B_o	=	Width of Bed Sediment Control Volume, feet
G	=	$\mathbf{q} \cdot \mathbf{B_o}$; The Sediment Discharge Rate by particle size, cubic feet/day
t	=	Computation time step, days
x	=	Length of Bed Sediment Control Volume, feet
y_s	=	Depth of Sediment Deposited or Eroded

The pair of equations are solved by numerical integration. The transport potential is calculated at the start of the computation time step using the sediment transport function selected for this simulation. It is not recalculated during the numerical integration steps, N = 1 to LTI. During the first iteration, N = 1, through the numerical integration of Event number 1, the fraction, PI(j), on the bed is the value which was calculated from the PF-Records as described above. However, according to Page 1 of ASCE Manual 54, Sedimentation embodies five processes:

erosion,

entrainment,

transportation,

deposition,

and the compaction of sediment.

HEC-6T is a computational, sedimentation model. Once the sediment transported out of the reach has been calculated for integration step N = 1, the numerator of the sediment continuity equation gives the amount of sediment, in tons by particle size, which either deposited onto or eroded off of the bed. In the case of the erosion/entrainment processes that weight is subtracted from the bed sediment reservoir by particle size class. In case of the deposition process, the weight is added to the bed sediment reservoir by particle size. Therefore, after the first integration step is completed and N becomes 2, new fractions, PI(j), for the bed layer are calculated. Consequently, in the non-equilibrium case, the transport capacity during integration step N=2 will be different from that during N=1 because the fraction, PI(j), is different.

Interpretion of HEC-6T Results

HEC-6T is an Initial-Boundary Value model. It takes the loads at the upstream end of the model, both water and sediment, and solves the equations to transfer the impact of those loads at all cross sections in the model. The solution is subject to the Base Level Control at the downstream end of the model. Several points need to be made about model behavior based on the input data discussed above.

1. If the Dmax on the PF-Record is less than the maximum particle size class on the I4-Record, the program will extend the PF curve to the maximum particle size on the I4-Record at a constant percentage of 100 percent.

2. The calculated transport of those particles larger than the maximun size on the PF-Records will be zero when PI() = 0. Consequently, all inflowing particles will deposit in the first reach.

3. T hose inflowing particles which are not present in the parent bed material will continue to deposit until the model establishes a bed gradation which will yield a transport capacity equal to the inflowing load for the hydraulic conditions. This is a natural consequence of non-equilibrium sediment transport.

4. When modeling a system which is out of equilibrium, it is crucial to develop a series of experiments which will demonstrate whether model results are reliable.

5. Often available data are so sparse that it is difficult to confirm a model to the non-equilibrium case. In such cases, locate a reach which is in equilibrium or locate a time when the stream was in equilibrium for model confirmation tests.

6. Sand and gravel sediment transport obeys the equilibrium principle. The equilibrium principle means that there is a relationship between the particle sizes in the bed sediment mixture and the concentrations in the flow field. The sand and gravel particles do not change the properties of the fluid. Therefore, for a given hydraulic energy condition, the concentration of sand and gravel in transport will reach a value such that the same number and size of particles are depositing as are eroding from the bed. When this condition occurs, the concentration being transported will neither increase nor decrease with distance. This relationship is provided by equilibrium sediment transport functions and the Einstein concept of sediment transportation. Such a relationship has not been demonstrated for cohesive sediment transport. These finer particles tend to modify the properties of the fluid leading to hyper-concentration flows and mud-flows

7. Major deposition or erosion of the bed profile at the upstream end of the model followed by no significant changes at downstream cross sections indicates that the inflowing sediment concentrations by particle size are not in equilibrium with the bed gradation in the model data for the calculated hydraulic forces in the simulation hydrograph. Model experiments must be devised to reveal whither such a condition is real or numerical due to inadequate data.

8. Model experiments should begin with simple hydrology. Start with a steady state water discharge. For the first test use the "channel forming water discharge." The channel forming water discharge will be that flow which if run for an infinitely long period of time will establish a channel which is in equilibrium with the boundary conditions and the hydraulic/sedimentation computations in the model. The test for equilibrium is a uniform sediment transport throughout the

model A reasonable water discharge to select will be about bank full in those reaches of the model which are in equilibrium.

9. When a change in the inflowing sediment load or the water discharge at the model boundary fails to change the bed profile along the entire length of the model, the length of time in the simulation hydrograph is too short. No change in the bed profile occurs only on a stream which is in equilibrium. (i.e. $\partial G / \partial X = 0$.)

10. Compare the reliability of the inflowing sediment concentrations by particle size to the reliability of the bed gradation data and choose which is the most reliable data set. Give emphasis to the most reliable data set when confirming a model.

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